Year 5 – 8 Continuity Work Group

Calculations with fractions

December 2018 to March 2019

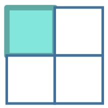
Facilitated by Tanya White
@MrsWhite1972

MathsHUBS
East Midlands West

Addition of fractions: a step-by-step approach

1. What is a fraction?

- What is one? Use pictorial and concrete representations

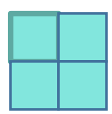
e.g.  1 ‘out of’ 4

- What is the numerator and denominator?

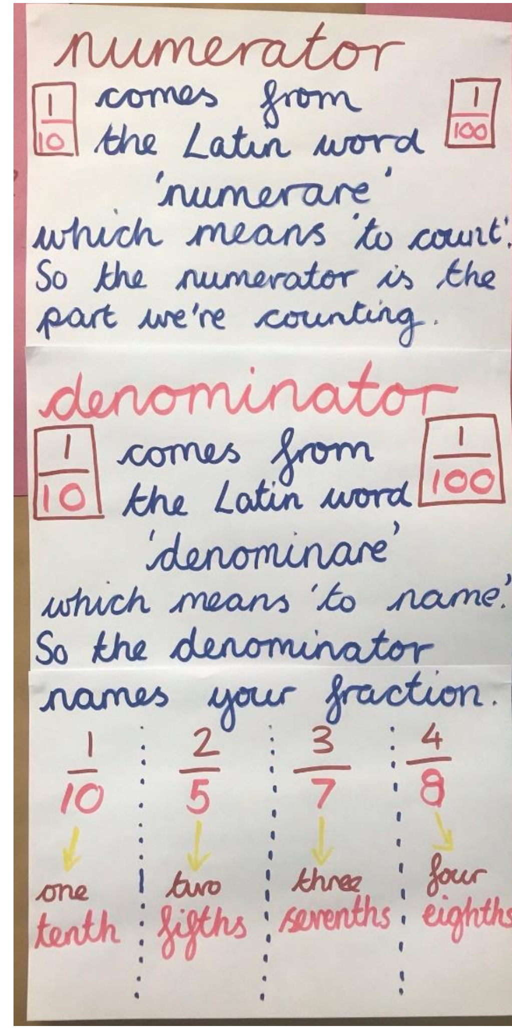
Refer to the Latin origins of the terms

- Whole number as a fraction

e.g. $5 = \frac{5}{1}$ $6 = \frac{6}{1}$ etc.

Also 4 out of 4 is the whole 

- Equivalence and simplifying (this needs to be thoroughly embedded knowledge)



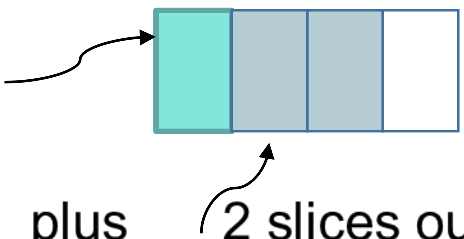
MathsHUBS East Midlands West

2. Adding same denominator using pictorial representations

- Why does the denominator stay the same?

Use example from a cake $\frac{1}{4}$ and $\frac{2}{4}$ are taken from the same cake, how much has been taken altogether is $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

1 slice out of 4



plus 2 slices out of 4

= 3 slices out of 4

3. Adding same denominator with numbers no simplifying

e.g. $\frac{1}{7} + \frac{2}{7} = \frac{3}{7}$

4. Adding same denominator with numbers and simplifying
(avoiding fractions that can simply be halved)

e.g. $\frac{1}{9} + \frac{2}{9} = \frac{3}{9}$ which can be simplified to $\frac{1}{3}$

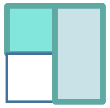
5. Lowest common multiple

- Spend enough time on this to ensure fluency

6. Adding different denominators only changing one – using pictorial representations

Back to the cake!

$$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

Quarter of the cake 

plus half of the cake

= three quarters of the cake

7. Adding different denominators only changing one using numbers

e.g. $\frac{1}{3} + \frac{5}{6} = \frac{7}{6}$ $only\ need\ to\ change\ the\ \frac{1}{3}\ to\ \frac{2}{6}$

8. Adding different denominators changing both fractions

- Revisit lowest common multiple and equivalent fractions

e.g. $\frac{1}{4} + \frac{1}{6} = ?$ $\frac{1}{4} = \frac{3}{12}\ and\ \frac{1}{6} = \frac{2}{12}$

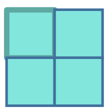
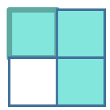
so $\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$

- Could show visually once equivalence has been found

9. Word problems involving addition of fractions

10. Improper and mixed fractions converting between the two

- Need to understand what the 1 represents in $1\frac{3}{4}$ for example

 and  $= \frac{7}{4}$

- Start with pictorial representations

11. Adding improper fractions with the same denominator

e.g. $\frac{5}{4} + \frac{6}{4} = \frac{11}{4}$

12. Adding improper fractions with different denominator

- Drawing on knowledge from step 8

13. Adding mixed numbers

- First with same denominators then with different
- Do you teach to change to improper fractions or not? Adding the whole number and then the fraction works well for addition but is not so straightforward for subtraction

14. Adding fractions to whole numbers

e.g. $\frac{3}{4} + 1 = ?$ $\frac{3}{4} + 5 = ?$

to check understanding of whole numbers as fractions

15. Word problems with addition of improper fractions and mixed numbers

Subtraction of fractions: a step-by-step approach

MathsHUBS East Midlands West

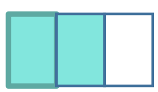
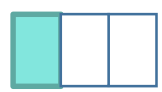
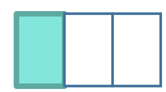
1. Counting in fractions

2. Understanding and comparing fractions

e.g. fraction fact families $\frac{3}{5} + \frac{2}{5} = 1$

3. Subtracting with the same denominator

- Using bar models (any denominator) or pizza model (small denominators)

$\frac{2}{3}$  subtract $\frac{1}{3}$  = $\frac{1}{3}$ 

MathsHUBS East Midlands West

4. Subtracting with different denominators

- Drawing on lowest common multiple work (Addition step 5)
- Using bar models to represent pictorially
- As with addition, start with only changing one denominator and move on to changing both denominators

e.g. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

5. Subtracting improper fractions then simplifying

e.g. $\frac{15}{6} - \frac{8}{6} = \frac{7}{6} = 1\ \frac{1}{6}$

6. Subtracting from a whole number

- This should draw on knowledge of fraction fact families as well as written calculations

e.g. $1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$ $\quad or \quad$ $3 - \frac{3}{4} = \frac{12}{4} - \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$

7. Subtracting using mixed numbers

- First convert to improper fractions

e.g. $4\frac{4}{9} - 1\frac{1}{3} = \frac{40}{9} - \frac{4}{3} = \frac{40}{9} - \frac{12}{9} = \frac{28}{9} = 3\frac{1}{9}$

8. **Word problems involving subtraction of fractions**

- Could also convert to decimals or percentages to solve the problems

9. **Introducing algebra into fractions**

e.g. $\frac{x}{6} - \frac{x}{7} = \frac{7x}{42} - \frac{6x}{42} = \frac{x}{42}$

MathsHUBS East Midlands West

Multiplication of fractions: a step-by-step approach

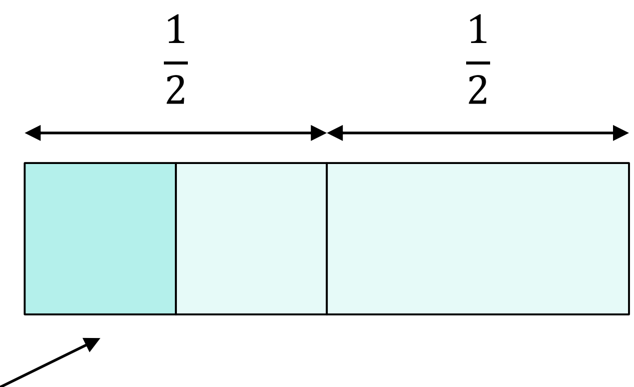
MathsHUBS East Midlands West

1. Spend time understanding the language of multiplication

- multiplication (x) → lots of → of e.g. $\frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{2}\ of\ \frac{1}{2}$

2. Find $\frac{1}{2}$ of various simple fractions

- Use the bar model to demonstrate



$$\frac{1}{2}\ of\ \frac{1}{2} = \frac{1}{4}$$

Now practise similar examples, e.g.:
$\frac{1}{2}\ of\ \frac{1}{4}, \frac{1}{5}, \frac{1}{7}$

3. Find $\frac{1}{3}$ of unit fractions

- Use the bar model first
- Can a rule be worked out?

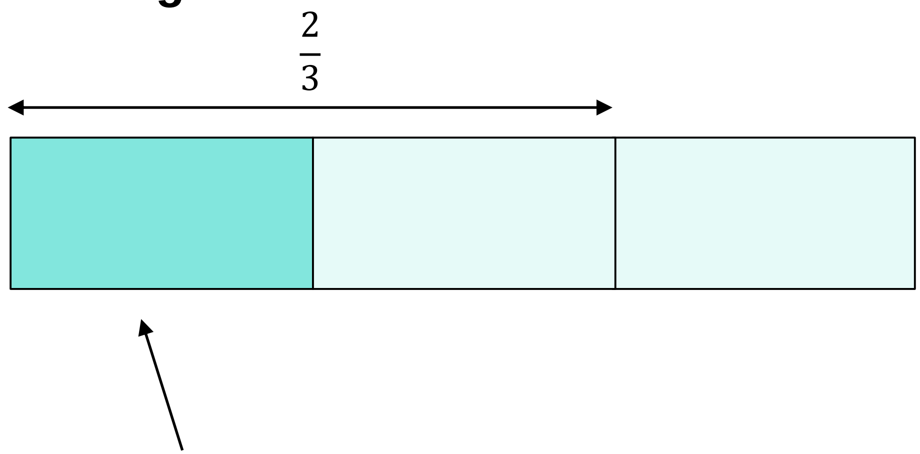
4. Move on to find different unit fractions of unit fractions

e.g. $\frac{1}{5}\ of\ \frac{1}{10}$

5. Lead into using the multiplication process once understanding is established

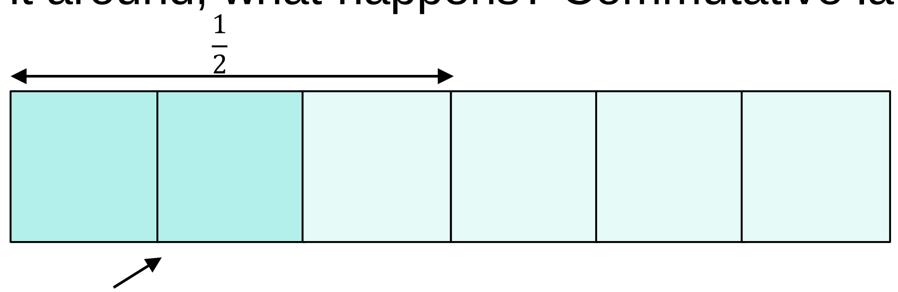
e.g. $\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$ by doing (1 x 1 = 1 and 2 x 8 = 16)

6. Now change the numerator



e.g. $\frac{1}{2}\ of\ \frac{2}{3} = \frac{1}{3}$

Swap it around, what happens? Commutative law



$$\frac{2}{3}\ of\ \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$$

7. Use multiplication process to multiply any number

e.g. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ by 4 x 2 = 8 and 5 x 3 = 15

8. Multiplying integers by fractions

- Understanding that 4 x $\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2}$ = 2

Or $\frac{1}{2} \times$ (of) 4 = 2

- Use the invisible denominator to lead into the multiplication process

e.g. 4 x $\frac{1}{2} = \frac{4}{1} \times \frac{1}{2} = \frac{4}{2}$ = 2

MathsHUBS East Midlands West

9. Multiplying integer by mixed number using partitioning

e.g. 10 x $2\frac{1}{2}$

- Solve by partitioning 10 x 2 = 20
  10 x $\frac{1}{2}$ = 5

  Then recombine 20 + 5 = 25

10. Multiplying integer by mixed number by converting to improper fractions

e.g. 10 x $2\frac{1}{2} = \frac{10}{1} \times \frac{5}{2} = \frac{50}{2} = 25$

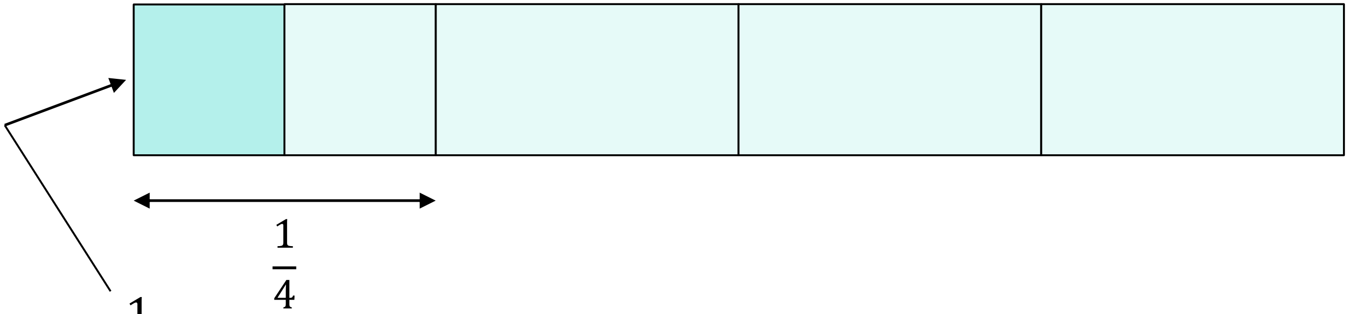
11. Multiplying mixed numbers by converting to improper fractions

e.g. $2\frac{1}{2} \times 3\ \frac{1}{3} = \frac{5}{2} \times \frac{10}{3} = \frac{50}{6} = \frac{25}{3} = 8\ \frac{1}{3}$

Division of fractions: a step-by-step approach

1. Division of fractions by whole numbers: start with dividing by 2

- Use a bar model

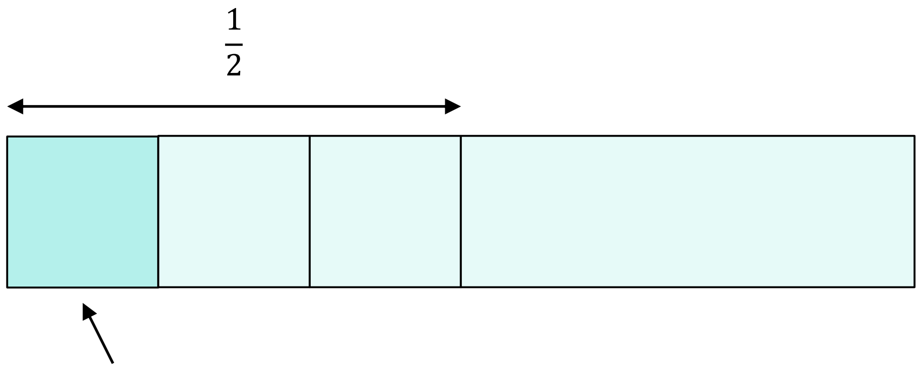


$$\frac{1}{4} \div 2 = \frac{1}{8}$$

- Practise with other unit fractions dividing by 2
- Practise with other fractions that can be drawn easily on a bar model dividing by 2

2. Division of fractions by whole numbers: dividing by integers other than 2

- Use a bar model



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

- Can the link between the fraction and the integer be spotted?

3. Division of fractions by whole number using keep, flip, change (KFC) method

e.g. $\frac{2}{5} \div 7 = \frac{2}{5} \div \frac{7}{1} = \frac{2}{5} \times \frac{1}{7} = \frac{2}{35}$

keep

change

flip

4. Whole number divided by a fraction: understand division as "goes into"

e.g.

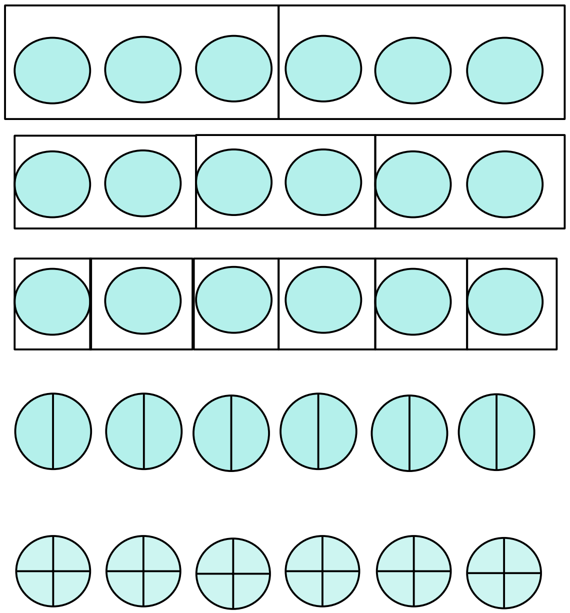
$6 \div 3 = 2$ = how many times does 3 go into 6

$6 \div 2 = 3$ = how many times does 2 go into 6

$6 \div 1 = 6$ = how many times does 1 go into 6

$6 \div \frac{1}{2} = 12$ = how many times does $\frac{1}{2}$ go into 6

$6 \div \frac{1}{4} = 24$ = how many times does $\frac{1}{4}$ go into 6



5. **Division of whole number by fraction using keep, flip, change (KFC) method**

e.g. $6 \div \frac{7}{10} = \frac{6}{1} \div \frac{7}{10} = \frac{6}{1} \times \frac{10}{7} = \frac{60}{7} = 8\,\frac{4}{7}$

keep change flip

6. Division of fractions by fractions using keep, flip, change (KFC) method

e.g. $\frac{4}{6} \div \frac{3}{7} = \frac{4}{6} \times \frac{7}{3} = \frac{28}{18} = \frac{14}{9} = 1\,\frac{5}{9}$

keep flip

change